

Psychometric functions for frequency discrimination from listeners with sensorineural hearing loss

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Psychometric functions for pulsed pure-tone frequency discrimination were obtained from hearing-impaired listeners at frequencies with normal hearing and at frequencies with mild or moderate hearing losses. The general form of psychometric functions at hearing-impaired frequencies was found to be the same as at normal-hearing frequencies, i.e., d' was linear with the frequency difference between tones, in Hz. For all but one psychometric function, the addition of an intercept term to the fitting equation did not account for significantly more variance than did the slope term alone. Therefore, it was concluded that psychometric functions for frequency discrimination can be adequately described with only one parameter: the slope of the psychometric function. Deficits in discrimination at hearing-loss frequencies were manifested by more gradual slopes of psychometric functions. Procedures for normalizing psychometric functions are presented, which facilitate comparisons of normal and impaired frequency discrimination data across studies and frequencies. Comparisons of dlf's (difference limen for frequency) obtained with adaptive and fixed procedures show a bias toward larger dlf's with adaptive procedures, but only at higher frequencies. A discussion of equal-interval and equal-ratio adaptive stepping rules indicates that an equal-ratio rule may be preferable.

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INTRODUCTION

Concerns about the form of psychometric functions for frequency discrimination and the appropriate metric for specifying sensitivity to frequency differences have both a theoretical and a practical rationale. Knowledge of the form of the psychometric function is important for appropriate applications of signal-detection models to frequency discrimination and for the design of maximally efficient and accurate adaptive procedures that estimate points on the psychometric function. On a more practical level, knowledge of the form of the psychometric function will facilitate cross-study comparisons of frequency discrimination obtained at different performance levels in different studies. For normal-hearing listeners, the form of the psychometric functions for frequency discrimination has been fairly well defined (Rabinowitz, 1970; Jesteadt and Sims, 1975; Turner and Nelson, 1982). These studies have shown that performance in a forced-choice pulsed pure-tone frequency discrimination task can be well described by a linear function relating the sensitivity index d' and frequency difference between standard and comparison tones, i.e., $d' = s(df)$, where the frequency difference between tones is given by df (or Δf) and the slope of the function is given by s .

The form of the psychometric function for frequency discrimination by hearing-impaired listeners has not received thorough study. Turner and Nelson (1982) published the psychometric function from one hearing-impaired

listener (their Fig. 1), which had the same general form as that for normal-hearing listeners, i.e., d' was a linear function of df with a zero intercept, but the slope of the function was drastically reduced. The implication of that finding is that the slope of the psychometric function is an appropriate index of sensitivity to frequency differences in the pulsed pure-tone frequency discrimination experiment. If so, only one point on the psychometric function need be determined to specify the entire psychometric function, and specification of frequency-discrimination deficits can be greatly simplified.

The main goal of the present investigation was to determine whether psychometric functions from hearing-impaired listeners can be adequately specified by a single sensitivity index. Before addressing that issue directly in this paper, we will examine techniques for normalizing psychometric functions across test frequencies in order to facilitate quantification of frequency-discrimination deficits. Finally, we present a preliminary comparison of dlf deficits and hearing losses, and examine the utility with which an adaptive procedure can estimate those deficits.

I. METHODS

Psychometric functions for frequency discrimination were obtained from ten ears of six listeners. Two of the ears, BI(L) and RA(L), had normal hearing throughout the entire frequency range. The remaining eight ears demonstrated mild to moderate hearing losses at one or more frequencies. Audiological findings in those ears were consistent with cochlear-hearing loss. In each hearing-impaired ear, an at-

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TABLE I. Summary of dlf deficits ($1/S$) specified by the sensitivity constant S from $d' = S(df/df_0)$, and adaptive-procedure biases specified as the ratio between adaptive dlf's [$dlf(a)$] and fixed dlf's [$dlf(f)$].

| Subj. | kHz | Threshold (dB HL) | dlf deficit | dlf(a) | | Subj. | kHz | Threshold (dB HL) | dlf deficit | dlf(a) | |
|-------|-----|-------------------|-------------|--------|--------|-------|-----|-------------------|-------------|--------|--------|
| | | | | dlf(f) | dlf(f) | | | | | dlf(f) | dlf(f) |
| RA(L) | 1.2 | 4 | 1.443 | 0.83 | DK(L) | 8.0 | -4 | 0.337 | 1.78 | | |
| DK(L) | 1.0 | 27 | 1.528 | 1.30 | DK(R) | 8.0 | 4 | 0.515 | 1.17 | | |
| BI(L) | 0.6 | -6 | 1.712 | 1.07 | BI(L) | 4.0 | 5 | 1.056 | 1.11 | | |
| RA(R) | 1.0 | 37 | 1.929 | 1.25 | RA(L) | 8.0 | 21 | 1.665 | 1.26 | | |
| LS(L) | 1.0 | 42 | 2.029 | 1.10 | DK(R) | 2.0 | 41 | 1.863 | 1.22 | | |
| MR(L) | 1.2 | 22 | 2.533 | 1.04 | LS(R) | 2.0 | 29 | 2.817 | 1.03 | | |
| LS(R) | 1.2 | 20 | 2.664 | 1.16 | LS(R) | 4.0 | 47 | 3.959 | 1.00 | | |
| MR(L) | 1.0 | 7 | 2.823 | 1.50 | MR(L) | 4.0 | 49 | 4.419 | 1.52 | | |
| MR(R) | 1.0 | 12 | 3.979 | 0.62 | LS(L) | 4.0 | 56 | 5.790 | 0.92 | | |
| EP(R) | 0.3 | 17 | 10.74 | 0.92 | MR(R) | 4.0 | 51 | 5.998 | 1.21 | | |
| EP(R) | 1.0 | 47 | 15.27 | 1.19 | EP(R) | 4.0 | 7 | 10.08 | 1.09 | | |

tempt was made to obtain psychometric functions from a region of normal-hearing or mild-hearing loss and from a region with moderate-hearing loss. Thresholds for each listener are given in Table I at the frequencies where frequency discrimination was measured. All listeners were well practiced on adaptive frequency-discrimination tasks at the time of participation in this experiment.

At the beginning of a test session, a four-alternative forced-choice (4AFC) adaptive threshold for frequency decrements was first obtained using an adaptive procedure identical to that described by Nelson *et al.* (1983), except that 16 reversals were used in threshold calculations instead of 8. The 4AFC adaptive threshold estimate produced an estimate of the df corresponding to 71% correct, which corresponds to a d' of 1.49 in a yes/no discrimination task (Elliott, 1964). With that estimate, and the assumption that the function is linear in d' per Hz with a zero intercept, four df 's were calculated that were expected to obtain performance levels corresponding to d' values of 2.0, 1.5, 1.0, and 0.5. Then 20 constant-stimulus 4AFC frequency-discrimination trials were collected at each df in descending order of df , followed by 20 more trials at each df in ascending order. This descending/ascending series continued until a total of 120 trials were presented at each of the four df conditions to obtain a single four-point psychometric function. Usually, psychometric functions at two test frequencies were obtained in this manner during a single listening session. During additional sessions, psychometric functions were collected at additional frequencies or, in some cases, psychometric functions obtained during previous sessions were repeated.¹

II. RESULTS AND DISCUSSION

A. Normalization of psychometric functions and dlf deficit

Figure 1 shows psychometric functions obtained from one hearing-impaired listener at three different test frequencies. Two characteristics of psychometric functions for frequency discrimination are demonstrated by this figure.

First, growth in performance, d' , appears to be adequately described as a linear function of df (in Hz), a premise that will be examined in some detail in the next section. Second, the slopes of these functions are dependent upon test frequency, the parameter of the graph. A third characteristic, not obvious in this type of plot, is the influence of hearing impairment on the slopes of these functions. On this type of graph, where none of the variables have been transformed, the effects of hearing impairment are not obvious because of the confounding effects of test frequency; both variables affect the slope of the psychometric function. Since our primary interest will eventually be to examine the influence of hearing loss on frequency discrimination, some way of factoring out performance differences due to test frequency is necessary so that the effects of hearing loss are foremost. This can be accomplished with a normalization procedure that transforms the independent variable, df , into units relative to "normal" frequency discrimination at some criterion performance level, as described by Zurek and Formby

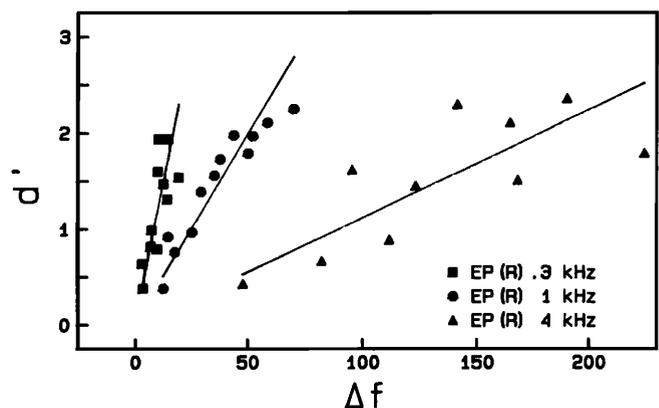


FIG. 1. Psychometric functions for frequency discrimination from one hearing-impaired listener at three different test frequencies. Solid lines are the best-fit functions to the equation $d' = s(df)$, where df is the frequency difference between standard and comparison tones in a 4AFC pulsed pure-tone frequency-discrimination paradigm, and s is the slope of the psychometric function (not normalized).

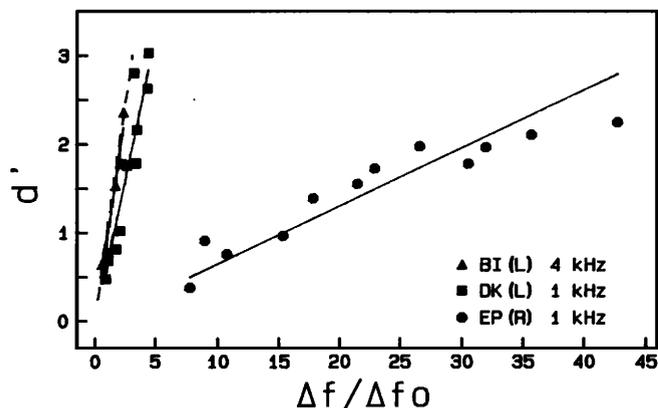


FIG. 2. Normalized psychometric functions for frequency discrimination from two hearing-impaired listeners, DK(L) and EP(R) at 1.0 kHz, and one normal-hearing listener, BI(L) at 4.0 kHz. Normalizing constants, df_0 , are estimates of "normal" df thresholds as a function of test frequency and test sensation level.² Solid lines are the best fitting linear functions through the origin. The dashed line is the predicted normal curve that has a slope (S) of 1.0 on these coordinates at all test frequencies.

(1981). Since preliminary estimates of normal frequency-discrimination thresholds at specific test frequencies and sensation levels (df_0) already exist (Harris, 1952; Wier *et al.*, 1977; Nelson *et al.*, 1983), a transformation into units relative to normal frequency discrimination is accomplished by $d' = S(df/df_0)$, where df_0 is the dlf expected from normal-hearing listeners at a specific frequency and at a specified performance level, and S is the slope of the transformed function. We used the Nelson *et al.* (1983) prediction equation, corrected to $d' = 1.0$, for our estimate of "normal" dlf's.²

An example of three psychometric functions, normalized according to these procedures, is shown in Fig. 2. We can see that a psychometric function that is normal will exhibit a slope, S , that is near unity, no matter what test frequency is used (as with subject BI at 4 kHz and subject DK at 1 kHz). The extent to which the slope of a function varies from unity (dashed line in Fig. 2) indicates the magnitude of deviation from normal performance. Poorer performance is indicated by more gradual slopes, as with subject EP at 1 kHz (the exact definition of normal performance requires appropriate dispersion statistics that are not yet available). It can be seen in Fig. 2 that this type of normalization procedure removes the effects of test frequency, leaving the effects of hearing impairment as the primary determinant of differences in slopes of normalized psychometric functions. Since this normalization procedure translates the normal slope to unity, the amount of relative impairment of frequency discrimination in hearing-impaired listeners is defined as $1/S$ and referred to as the dlf deficit.

Specification of frequency-discrimination deficits in terms of slope changes, as shown in Fig. 2, has not been a common practice, and might not be intuitively appealing to some readers. On the other hand, specification of abnormal frequency discrimination as an increase in the dlf has been a common way of expressing deficits in frequency discrimination, and can be easily accomplished by taking the logarithm

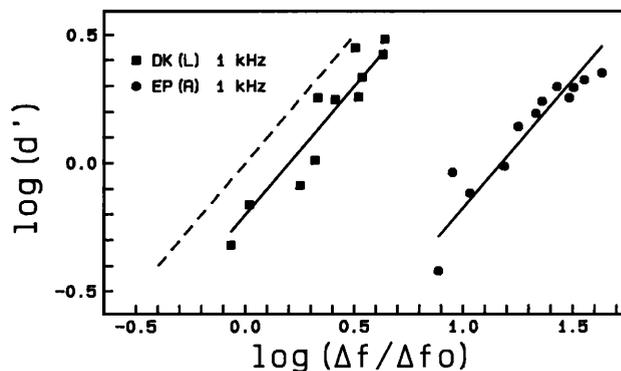


FIG. 3. Normalized psychometric functions on log-log coordinates, which demonstrate dlf deficits as horizontal shifts along a linear dlf deficit scale. All curves have unit slopes, and dlf deficits are represented by functions that are shifted to the right. The dashed line is the normal function. The solid lines are the "best-fit" curves to the equation $\log(d') = \log(S) + \log(df/df_0)$.

mic transform of the linear function, $d' = S(df/df_0)$, i.e., $\log(d') = \log(S) + \log(df/df_0)$. With this transformation, dlf deficits can be displayed as horizontal shifts of psychometric functions along the x axis, a display that is similar to that used to describe deficits in hearing sensitivity for intensity. The transform yields a linear psychometric function in log-log coordinates with a slope of 1.0 and an intercept that specifies the dlf deficit. The negative of that intercept, $-\log(S)$ or $\log(1/S)$, is the displacement of the psychometric function along the x axis, which is the dlf deficit in log units.

This transformation is illustrated in Fig. 3 for two psychometric functions at 1 kHz, one from a listener with a function close to normal and the other from a listener with a function that is more than fifteen times normal. The near normal function from DK(L) is displaced only slightly to the right along the x axis, by 0.18 log units, which is a dlf deficit factor of only 1.53 (antilog of 0.18). By contrast, the function from EP(R) is greatly displaced to the right of the normal curve, by 1.18 log units, which is a dlf deficit of 15.3. At any particular level of performance, listener EP(R) requires frequency differences that are fifteen times those required by a normal-hearing listener. This log-log transform linearizes the data so that equal changes in the independent variable, expressed as the logarithm of dlf deficit, result in equal changes in the dependent variable, expressed as the logarithm of performance. This type of transformation facilitates not only comparisons among dlf deficits, independent of test frequency, but also, as we shall see later, comparisons between dlf deficits and amount of hearing loss.

B. Contribution of the Intercept

Of primary interest in this research was the appropriate form of the equation used to represent psychometric functions for frequency discrimination. Specifically, we wished to determine if the simple linear function, $d' = S(df/df_0)$, is adequate to specify psychometric functions at normal-hearing and at hearing-loss frequencies, or, as with certain fre-

quency-modulation paradigms (Jesteadt and Sims, 1975), whether an additional term is necessary to account for a nonzero intercept, as in $d' = S(df/df_0) + k$.

Evidence of the adequacy of the single-term fits to the data was obtained by comparing the amount of variance accounted for by the slope term alone, as in $d' = S(df/df_0)$, with the amount of variance accounted for by the addition of an intercept term, as in $d' = S(df/df_0) + k$. Regression analysis, consisting of two linear fits to each psychometric function, was performed on the data.³ The first fit constrained the intercepts to be zero and only the slopes were determined. The second fit allowed both the slopes and intercepts of the functions to vary. Analyses of variance were performed to determine whether or not the use of an intercept accounted for significantly more variance than did the slope alone (Torrie and Steel, 1980).

Out of the 22 psychometric functions evaluated, only one demonstrated that the addition of the intercept term accounted for significantly ($p < 0.01$) more variance than the slope term alone. In that case the psychometric function was only based upon three points, making it questionable to include it in the analysis at all. These results provide strong evidence that the simple linear function, $d' = S(df/df_0)$, can adequately describe psychometric functions for frequency discrimination at both normal-hearing and hearing-impaired frequencies, for test frequencies ranging from 0.3 to 8.0 kHz. This conclusion could also be generalized to a range of sensation levels, since the sensation levels tested here ranged from 33 dB up to 70 dB. It would appear that a simple multiplicative constant, the slope of the psychometric function, is a sufficient index of sensitivity to frequency changes

in the impaired auditory system, at least as measured with this pulsed pure-tone frequency-discrimination paradigm, and for "cochlear" type hearing losses.

C. The dlf deficit and hearing loss

With some certainty about the appropriate metric for specifying frequency-discrimination performance in hearing-impaired listeners, a preliminary examination can be made of the relation between dlf deficits and hearing loss at the test frequency. Also, dlf deficits, $1/S$, were calculated from the S values obtained from the single-term fits in the previous regression analysis and are listed in Table I along with the amount of hearing loss at each frequency. The relations between dlf deficit and hearing loss can be seen in Fig. 4, where dlf deficits are plotted on the abscissa using the same logarithmic scale shown in Fig. 3. In this case, the logarithmic transform of dlf deficit was chosen because it provided better linear fits than a simple linear representation of dlf deficit. It is also the scale used earlier by Zurek and Formby (1981) to make similar comparisons.

Certain aspects of these results can be compared with those of the Zurek and Formby (1981) study, in which hearing-impaired listeners detected frequency modulation. For the frequencies from 2–8 kHz, shown by the filled triangles in Fig. 4, there is a strong positive relation between dlf deficit and hearing loss. The solid line shows the least-squares fit to the data, indicating a positive relation that has a correlation coefficient of 0.95.⁴ Over a 60-dB range of hearing level, dlf deficit varies by a factor of 13. This is consistent with Zurek and Formby's data for frequencies from 0.5–4 kHz, although their subjects did not begin to show sizable deficits unless they had more than 40 dB of hearing loss at the test frequency. For the frequencies from 0.6–1.2 kHz, shown by the filled circles in Fig. 4, there is not a strong relation between dlf deficit and hearing loss. The dashed line shows the least-squares fit to the data, which has a correlation coefficient of -0.03 . This is also consistent with Zurek and Formby's data, since at comparable frequencies none of their subjects whose hearing losses were less than 40 dB demonstrated large dlf deficits, and none of our subjects had hearing losses greater than 40 dB at these frequencies.

We regard this type of comparison of the relation between dlf deficit and hearing loss as somewhat tentative, since the range of hearing losses is small, the number of frequencies tested per ear are few, and the numbers and types of hearing-impaired listeners are limited. Furthermore, not all previous studies of frequency discrimination in hearing-impaired listeners have demonstrated strong relations between hearing loss and frequency-discrimination performance (Tyler *et al.*, 1983). However, this tentative comparison does demonstrate the type of trends we might expect in a larger study. It also indicates that we can expect some large individual differences. For example, consider the data shown in Fig. 4 by the open symbols, which were not included in the above analysis. These are the results for the hearing-impaired listener EP(R). Although this listener had only mild hearing losses at 0.3 kHz (open circle) and 4 kHz (open triangle), she demonstrated dlf deficits greater than 10.0 at those frequencies. Other listeners with more hearing

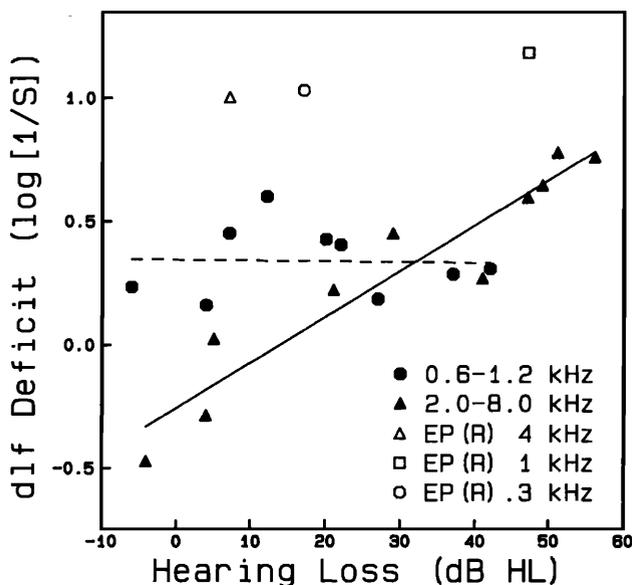


FIG. 4. Comparisons of dlf deficits as a function of the amount of hearing loss at each test frequency. The dlf deficits, $1/S$, were calculated with the linear equation $d' = S(df/df_0)$. Data from all 22 psychometric functions are shown. Filled circles and the dashed line are for test frequencies from 0.6–1.2 kHz, filled triangles and the solid line are for frequencies from 2.0–8.0 kHz. The open symbols show dlf deficit estimates from a hearing-impaired listener who appeared to be different from the rest of the hearing-impaired listeners (see text).

loss performed as well as normals. This listener is also unusual because her dlf deficit of 15.3 at 1 kHz (open square) is so much larger than the other listeners who had nearly as much hearing loss. These results suggest that the physiological mechanisms responsible for losses in hearing sensitivity may not necessarily be the same as those responsible for dlf deficits (Turner and Nelson, 1982; Tyler *et al.*, 1983).

D. Fixed versus adaptive procedures

Since our data collection procedures utilized an adaptive procedure to first estimate a df corresponding to a d' of 1.49 and then immediately obtained a four-point psychometric function with a fixed procedure, we can look for evidence of consistent bias in the adaptive procedure and can also determine the estimation accuracy of the adaptive procedure.

The first obvious comparison is to determine how well the adaptive procedure estimated the range of df 's that were to be used in the fixed procedure, to see if there were any consistent biases. This can be done by comparing the d' values expected on the basis of the adaptive dlf with the d' values actually obtained with the fixed procedure. This comparison, based on the average performance across subjects, is shown in Fig. 5 for signals in two different frequency regions. For each test ear and frequency, the expected d' values were always the same, since df 's expected to yield d' values of 0.5, 1.0, 1.5, and 2.0 were calculated from each adaptive dlf. The obtained d' values are the averages of the d' values obtained across retests at each of those df 's. A perfect relationship is shown by the solid line with zero intercept and unit slope. For the low-frequency signals (0.3–1.2 kHz) there was excellent agreement between expected and obtained performance. Using a t statistic, we found no significant differences ($p > 0.05$) between the expected and obtained mean d' values. The lack of a bias for low frequencies is demonstrated by

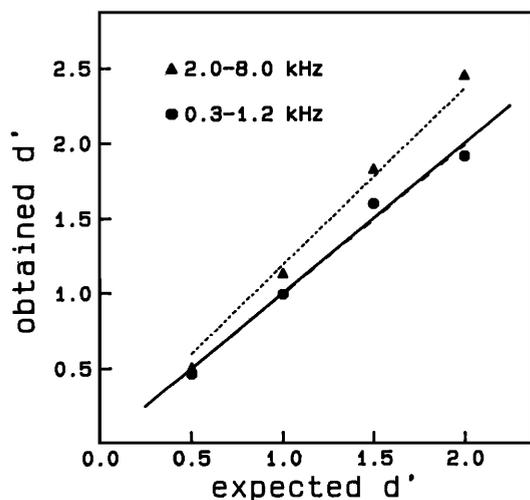


FIG. 5. Comparisons of d' values obtained in the fixed procedure with d' values expected from the dlf that was obtained in the adaptive procedure. The solid line shows a perfect relationship. The dashed line is the fit through zero for the low frequencies, which shows no consistent bias. The dotted line is the fit through zero for high frequencies, which shows a consistent bias toward better performance with the fixed procedure, by a factor of 1.2, than expected from the adaptive dlf estimate.

the linear least-squares fit through zero (dashed line) that has a slope of 1.0. However, for the high-frequency signals (2.0–8.0 kHz) there was a strong bias toward better performance in the fixed procedure than predicted from the adaptive procedure at the two largest d' values ($p < 0.025$). The trend is toward larger biases at larger values of d' . The amount of bias is given by the slope of the linear least-squares fit through zero (dotted line), which is 1.2. This means that, to a first approximation, performance obtained with the fixed procedure was better, by a factor of 1.2, than the performance expected from the adaptive procedure. This also means that the adaptive dlf's should be larger than the fixed dlf's by a factor of 1.2, as is confirmed below.

A different comparison between the results of the adaptive and fixed procedure provides information about the estimation accuracy of the quicker adaptive procedure. This can be done by comparing the final dlf's obtained from the adaptive procedure (adaptive dlf) with the final dlf's obtained from the psychometric function (fixed dlf). The final adaptive dlf was taken as the mean adaptive dlf across retests; the final fixed dlf was specified by the dlf at $d' = 1.49$ from the linear least-squares fits through zero of the psychometric functions. Table I gives the ratios of those two estimates for each ear and test frequency, and Fig. 6 compares those two estimates on log-log coordinates. Again, only for the high-frequency dlf's (2.0–8.0 kHz), shown as filled triangles in Fig. 6, was there a significant average bias (factor of 1.2) toward larger dlf's with the adaptive procedure than with the fixed procedure ($p < 0.01$). The trend of this bias was the same, and significant, when comparisons were made using single-run data without averaging across retests.

The reason for this bias is not clear. Initially, we reasoned that because we used equal-interval step sizes that were proportional to test frequency (0.12%), this might have led to step sizes that were too small in terms of changes in performance expected from each adaptive step (d'/step),

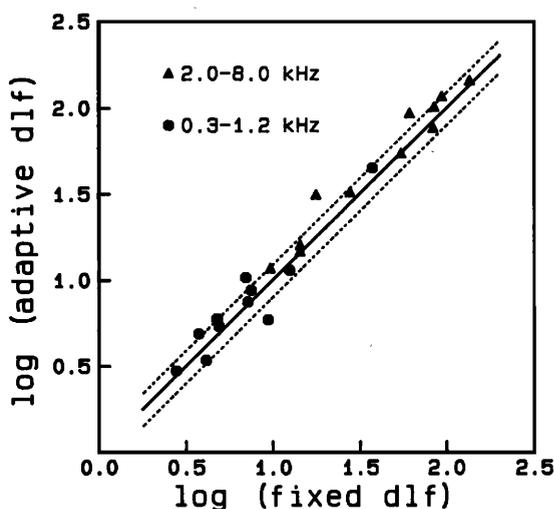


FIG. 6. Comparison of dlf's (df at $d' = 1.49$) obtained in the adaptive procedures with dlf's obtained in the fixed procedures. Low frequencies (0.6–1.2 kHz) are shown by the filled circles, high frequencies are shown by the filled triangles. The solid line is the perfect relationship. Dashed lines show the standard deviation of individual bias factors, i.e., 68% of the adaptive dlf's fell within a factor of 1.24 of the fixed dlf.

especially at high frequencies and in impaired ears with large dlf deficits. The use of overly small step sizes might have led to frustration, causing a subject to "give up" during a tracking run and thereby provide us with a premature estimate of threshold. However, a careful examination of the d' /step used to obtain the adaptive dlf's, across individual ears and test frequencies, indicated that the biases were not correlated with d' /step. Therefore, the reason for this bias remains unexplained.

Another characteristic of estimation accuracy that should be considered is the dispersion of bias factors between adaptive and fixed dlf's. That dispersion can be represented by the standard deviation of individual bias factors, which we call a dispersion factor. The dispersion factor for the high frequencies (1.22) was only slightly less than for the lower frequencies (1.27), with a dispersion factor of 1.24 across all test frequencies. This is shown in Fig. 6 as one standard deviation above and below (dotted lines) the exact relationship (solid line). These dispersion data indicate that one can expect 68% of adaptive dlf's, across all test frequencies, to be within a factor of 1.24 of fixed dlf's. Considering that the adaptive dlf was estimated in about one-tenth the number of trials it took to obtain a four-point psychometric function, these dispersion factors seem acceptable for most dlf experiments, especially considering that across-subject dispersion factors can be considerably larger (Nelson *et al.*, 1983).

The equal-interval stepping rule used here to obtain adaptive dlf's (step size was 0.12% of test frequency) insured that a symmetrical distribution of df 's along the psychometric function were sampled during an adaptive run, as opposed to equal-ratio stepping rules (as in Wier *et al.*, 1977) that result in asymmetrical (geometric) distributions. We cannot argue that the symmetry or asymmetry of the sampling distribution of df 's have obvious biasing effects because the target performance level estimated in an adaptive procedure is primarily determined by the stepping rule (Levitt, 1971). However, as we analyzed d' /step values, looking for an explanation for the high-frequency bias, it became clear to us that our use of equal-interval step sizes led to sampling-distribution densities that were considerably greater in listeners with larger dlf deficits. The differences were on the order of 10:1. For example, for DK (L) at 1.0 kHz the d' /step ratio was 0.50, while for EP (R) at 1.0 kHz it was 0.05. These would translate into 5.8 df adaptive steps for DK and 58 steps for EP between d' values of 0.3 and 3.0, which is essentially the useful range for specifying psychometric functions. Clearly, sampling the psychometric function in 58 equal-ratio steps is inefficient.

If, on the other hand, an equal-ratio stepping rule had been employed, then the distribution densities would have been independent of dlf deficit. Although the sampling distributions would have been asymmetrical (geometric), distribution density would have been the same for every psychometric function. This can be demonstrated more clearly using the log-log coordinates of Fig. 3. Here one can see that equal-ratio changes in df/df_0 (indicated by equidistant points along the abscissa) correspond to equal-ratio changes in d' (indicated by equidistant points along the ordinate), independent of which psychometric function is sam-

pled. If one were to choose an equal-ratio step factor of 2.0, as in the Wier *et al.* (1977) study, the number of df steps covering a 10:1 range in performance along the psychometric function, i.e., from a d' of 0.3 to 3.0, would be 3.32 $[1/\log(2.0)]$ for both of the psychometric functions shown in Fig. 3. Intuitively, it would seem more appropriate to utilize an equal-ratio step factor, both to increase adaptive efficiency and to insure equal sampling-distribution densities across subjects and frequencies. If 3.32 steps per psychometric function is judged to be too gross a sampling density, one might use a step factor of 2.0 for the first few adaptive reversals and then switch to a smaller step factor, as per Freyman and Nelson (1986a, 1986b). They used a final step factor of 1.19, which is a sampling density of 13.2 points per psychometric function. In addition, with the asymmetrical (geometric) sampling distribution resulting from an equal-ratio stepping rule, it would seem more appropriate to use, as they did, a geometric rather than an arithmetic average of adaptive df reversals for an estimate of threshold.

III. CONCLUSIONS

An examination of psychometric functions for frequency discrimination from frequency regions of normal- and impaired-hearing sensitivity indicates that the slope of the psychometric function is a sufficient indicator of sensitivity to frequency differences in both cases. The addition of an additive constant to the linear function $d' = S(df/df_0)$ does not account for significantly more variance. Normalization of psychometric functions allows the comparison of psychometric functions across frequencies and individuals, and dlf deficits take the form $1/S$. Additionally, the logarithmic transform of that function allows performance deficits to be displayed as horizontal shifts in psychometric functions along the x axis, and it provides for a better linear relationship when comparing dlf deficits and hearing losses.

A preliminary examination of the relation between the logarithm of dlf deficit and hearing loss at the test frequency confirms previous findings of a strong positive correlation at high frequencies, but also indicates that consistent individual differences are to be expected from a larger sample of hearing-impaired listeners, differences that imply different underlying mechanisms for sensitivity loss and dlf deficit. A comparison of adaptive and fixed dlf's indicates that the adaptive dlf at high frequencies was biased by a factor of 1.2 over the fixed dlf, a result not adequately explained. Careful examination of adaptive df sampling distributions and distribution densities indicated that an equal-ratio step factor, apart from being more efficient than an equal-interval step size, will insure equivalent distribution densities across subjects and test frequencies.

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¹Whenever four-point psychometric functions were repeated, the initial adaptive df thresholds were also repeated. Those retests, of course, yielded slightly different adaptive df thresholds. Since retest adaptive df thresholds were used to calculate the four df 's for the retests of the psychometric function, the exact df 's used on repeated psychometric functions also differed. In those cases, all data points were included in the final psychometric function, each point representing performance on 120-discrimination trials. This is seen in Fig. 1, where more than four points are included in each psychometric function.

²To accomplish normalization of the data shown here, estimates of normal frequency-discrimination thresholds at different frequencies and at different sensation levels, df_0 , were specified by the general prediction equation in Nelson *et al.* (1983) using the constants obtained from the Harris (1952) data. The constants in that equation predict frequency discrimination thresholds at $d' = 1.49$. Since we wished to normalize the data to a performance level of $d' = 1.0$, the equation predictions were divided by 1.49. Therefore, the normal psychometric function in Fig. 2 has a slope of 1.0. Since the sensation levels at which frequency discrimination was tested were all above 30 dB at hearing-impaired frequencies and above 40 dB at normal-hearing frequencies, and frequency-discrimination thresholds are relatively constant above these sensation levels, differences in sensation level would have very little effect on psychometric functions. However, in the interest of completeness, exact sensation levels of the test stimuli were included in the parameters of the general prediction equation used to calculate df_0 values for the normalization of each psychometric function.

³At large values of df some of the psychometric functions tend to "saturate," i.e., no matter how large the df , there is a tendency for d' to be limited by subjects inadvertently pushing the wrong response key or by momentary lapses in attention (Rabinowitz *et al.*, 1976). Therefore, for the psychometric functions in this study, d' values were limited to 3.2 or less.

⁴Note that the solid line has a negative intercept for the frequencies 2–8 kHz. At 8 kHz, two dlf deficits were 0.34 and 0.51. This indicates that the normative data used to estimate df_0 may not have been sensitive enough and these two dlf's simply represent one extreme of the normal distribution, or that these examples of hyperacuity are manifestations of some unusual auditory pathology. Since frequency-discrimination norms and appropriate dispersion statistics for those norms have yet to be determined, we cannot say which.

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