A general equation describing frequency discrimination as a function of frequency and sensation level

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Frequency-discrimination thresholds, for a wide range of stimulus frequencies and stimulus levels, were obtained from three normal-hearing listeners. Linear regression analyses of the present data, and of data from two previous studies, indicate that an SL^{-1} transformation of stimulus level and a $\sqrt{F}$ transformation of stimulus frequency yield linear dimensions that allow accurate predictions of frequency-discrimination thresholds from normal-hearing listeners over a wide range of stimulus frequencies (125–8000 Hz) and stimulus levels (5–80 dB SL) with a single prediction equation.

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INTRODUCTION

Wier *et al.* (1977) obtained frequency discrimination thresholds from normal-hearing listeners over a broad range of signal levels and test frequencies. Their results, perhaps the most comprehensive since Shower and Biddulph (1931), were well described by a linear equation at each sensation level, the parameters of which varied with sensation level. The implication of their findings is that frequency discrimination in normal-hearing listeners, over a broad range of intensities and frequencies, might be specified by a single equation. This research replicates the Wier, Jesteadt, and Green study, but uses a four-interval instead of a two-interval forced-choice procedure, a linear adaptive step size rather than a ratio step size, and does not employ a background noise during data collection as they did. It then examines the accuracy with which a general prediction equation can describe frequency discrimination data from normal-hearing listeners. This is done for data from the present study, for data from the Wier *et al.* (1977) study, and for data from an earlier study by Harris (1952).

I. METHOD

Pure-tone test signals were generated by a programmable oscillator (Krohn-Hite 4141R) that produced sine waves with harmonic-distortion components more than 70 dB below the fundamental. Signals were gated by an electronic switch using rise and decay times of 10 msec. Signal duration was 300 msec. The intensities of test signals were controlled by programmable attenuators. Signals were presented monaurally in a TDH-49 earphone mounted in an MX-41/AR cushion. Listeners were seated in a double-walled sound treated booth.

A four-interval four-alternative forced choice (4AFC) procedure was used. Each 4AFC trial consisted of a warning interval of 500 msec, followed by four listening intervals of 500 msec and an answer interval; an appropriate light indicated each interval. During three of the four listening intervals, 300-msec tone bursts at the standard frequency were presented; during one of the four intervals, chosen randomly from trial to trial, a 300-msec tone burst at the variable frequency was presented. The variable frequency was always less than or equal to the standard frequency, i.e., the task involved frequency-decrement detection. A 200-msec silent period followed each tone burst. Standard frequencies were at 0.3, 0.6, 1.2, 2.0, 4.0, and 8.0 kHz. Visual feedback of the correct interval followed each response.

An adaptive procedure was used to estimate 71% correct performance (Levitt, 1971). It utilized a two-up, one-down decision rule and final step sizes that were 0.12% of the standard frequencies (rounded to the nearest 0.1 Hz). Determination of each frequency-difference threshold began with an initial frequency difference that was 3% of the standard frequency and an initial step size that was 0.6% of the standard frequency. After three reversals, the step size was reduced to 0.12% of the standard frequency. Thresholds were calculated as the average of the frequency differences that existed on the last 8 out of 12 threshold-tracking reversals. On the trials used to estimate frequency-difference thresholds, one incorrect response resulted in the choice of a larger frequency difference for the next trial, and two correct responses resulted in the choice of a smaller frequency difference for the next trial.

Three normal-hearing listeners participated. None were musicians. Two had extensive experience with frequency-discrimination tasks at 1200 Hz prior to this experiment (MS and AT), one did not (BI). Before frequency-discrimination testing began, sensitivity thresholds were obtained in quiet from each listener using a 4AFC adaptive procedure that employed 2-dB step sizes and a two-up, one-down stepping rule. Those thresholds are given in Table I. As in the Wier *et al.* (1977) study, quiet thresholds were consistent with the ANSI (1969) norms and with data obtained by rigid psychophysical procedures in other laboratories (Watson *et al.*, 1972).

During each test session, frequency-discrimination thresholds were obtained for one of six test frequencies at signal levels that ranged from 80 dB SPL to 10 dB SPL in decreasing steps of 10 dB. Four threshold determinations were made at each intensity and frequency for listeners AT and MS. Listener BI was tested five times in every condition, but only the last four measures were used in the data analyses.

TABLE I. Quiet thresholds obtained with the 4AFC adaptive procedure. Threshold values are the means of four retests expressed in dB SPL. Values enclosed within parentheses are thresholds in dB HL (*re*: ANSI 1969).

Listener	Frequency (Hz) 300	600	1200	2000	4000	8000
MS (LE)	26.7	12.7	8.7	7.7	14.9	18.0
	(3.7)	(0.7)	(0.7)	(−3.3)	(4.4)	(4.5)
AT (RE)	28.2	9.2	5.2	1.5	11.5	4.3
	(5.2)	(−2.8)	(−2.8)	(−9.5)	(1.0)	(−9.2)
BI (LE)	22.7	6.5	10.2	12.5	15.5	16.5
	(−0.3)	(−5.5)	(2.2)	(1.5)	(5.0)	(3.0)

II. RESULTS AND DISCUSSION

A. Stimulus intensity

Results from the three listeners are shown in Fig. 1. The difference limen for frequency (DLF) is plotted on a logarithmic scale as a function of stimulus intensity. For convenience, these curves are called DLF intensity functions. As in previous studies (Shower and Biddulph, 1931; Harris, 1952; Wier *et al.*, 1977), the DLFs in Fig. 1 are decreasing functions of signal intensity, which tend to asymptote at moderate signal levels. To demonstrate the variability that exists from one retest to another, four DLF intensity functions, each obtained during separate listening sessions, are shown for each listener.

In order to make direct comparisons between our results and those from other studies, certain transformations of the data were necessary. First, the independent variable was transformed from SPL into sensation level (SL), using

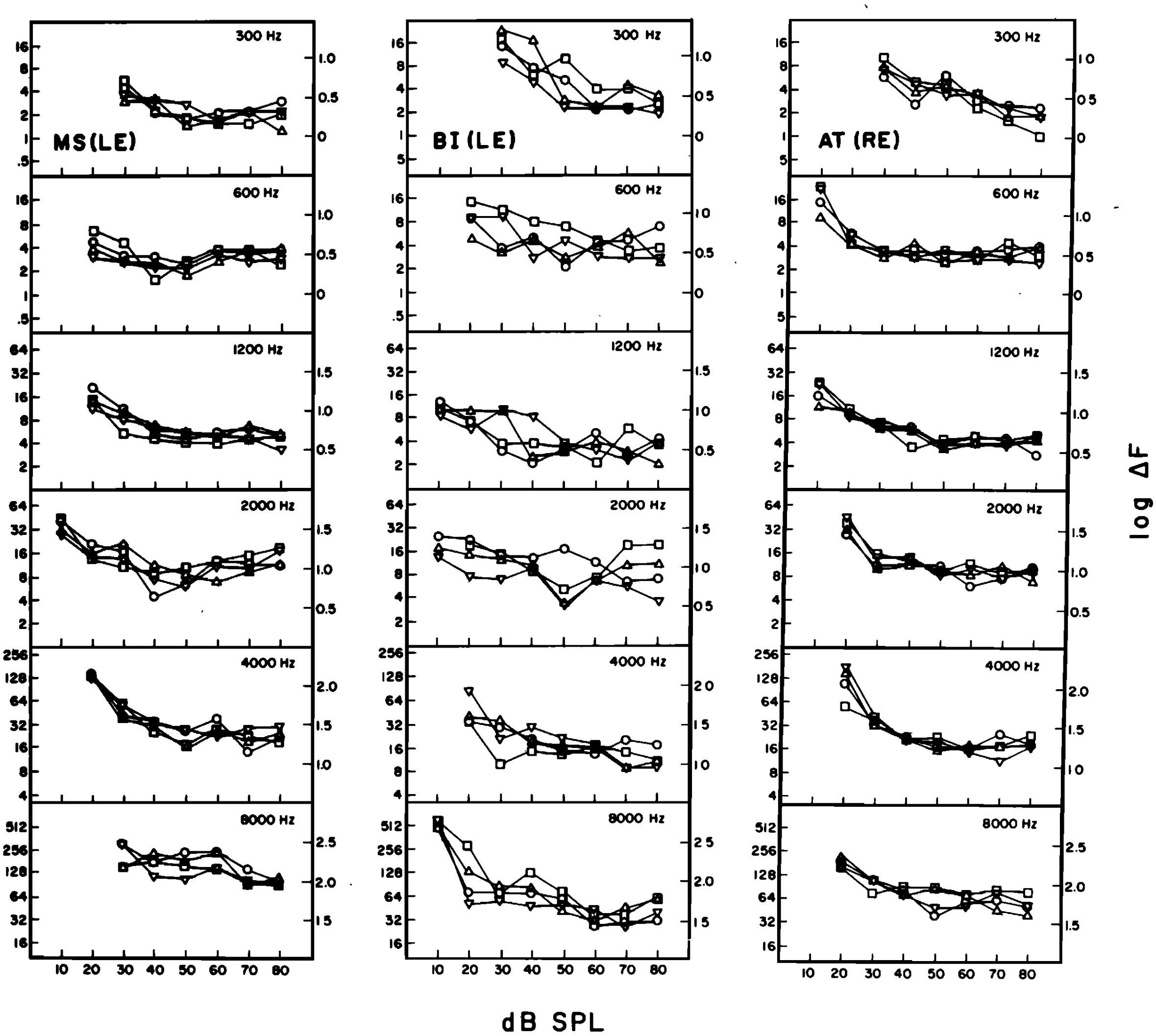

FIG. 1. Individual DLF intensity functions from three normal-hearing listeners. Each symbol represents a frequency-difference threshold obtained at a single sitting. Retest order is indicated as follows: 1st-circles, 2nd-squares, 3rd-upward triangles, 4th-downward triangles.

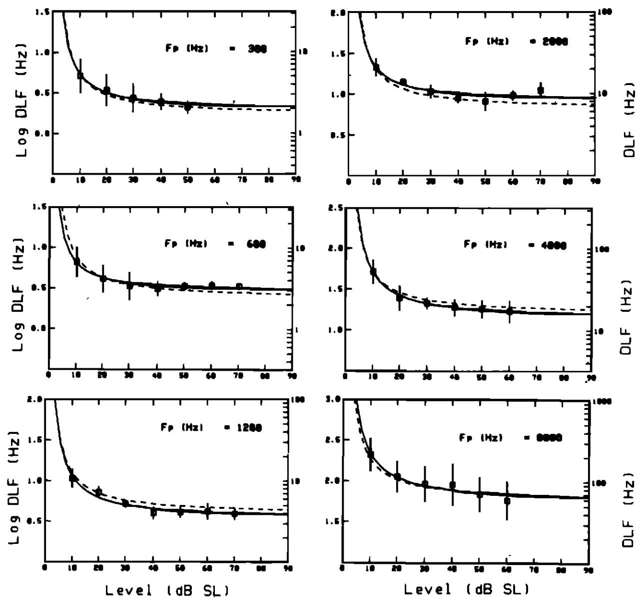

FIG. 2. Averaged DLF intensity functions from the three normal-hearing listeners in the present study. The independent variable of Fig. 1 was converted to sensation level (SL) and interpolations were made to the nearest SL decade. The solid curves are the fits of each averaged DLF intensity function using Eq. (1). Dashed lines show the DLFs predicted by the general prediction equation, Eq. (4), using the constants given in Table V.

the average 4AFC sensitivity threshold obtained from each listener at each test frequency in quiet. Then interpolations were made to the nearest SL decade. This was done so that the DLFs could be averaged across listeners at equal SLs; it would also allow eventual comparisons to be made at the same SLs used by previous investigators. The results of those transformations are shown in Fig. 2. Each point is the geometric mean, across subjects, of the interpolated DLFs. Vertical lines indicate one standard deviation above and below each mean.

Since one of the goals of this investigation was to attempt a general description of DLFs over a broad range of frequencies and intensities, the data in Fig. 2 were subjected to various least-squares fitting procedures, looking for a simple equation that would minimize the error between predicted and actual DLFs. After exploring several different transforms of signal level, which would "linearize" the DLF intensity functions sufficiently to achieve a good least-squares fit to the individual as well as to the group data, we settled on the transform given by Eq. (1), where SL is the sensation level of the test signal in decibels. Least-squares fits to mean DLFs with Eq. (1) are shown by the solid curves in Fig. 2. Coefficients of correlation for those fits are given in Table II. Equation (1) does quite well at describing group mean DLF intensity functions. At all frequencies but 2000 Hz, Eq. (1) accounts for more than 93% of the variance; at 2000 Hz it accounts for 86% of the variance. With a couple of exceptions for listener MS, at 600 and 2000 Hz, Eq. (1) also describes DLF intensity functions from individual listeners quite well.

$$\log \mathrm{DLF} = k + m(\mathrm{SL}^{-1}) . \quad (1)$$

Table II also shows the results of the least-squares analysis using Eq. (1) on data from the Wier *et al.* (1977) study. Their DLFs were corrected for differences in performance levels between the two studies. Wier *et al.* used a 2AFC procedure in which 71% correct is equivalent to a d' of 0.78 in a yes–no procedure; the present study used a 4AFC procedure in which 71% correct is equivalent to a d' of 1.49 in a yes–no procedure (Swets, 1964). Since the form of the psychometric function for frequency discrimination has been shown to be

TABLE II. Coefficients of correlation (r^2) from the least-squares analysis of DLF intensity functions using Eq. (1).

Nelson, Stanton, and Freyman

	Frequency (Hz)					
	300	600	1200	2000	4000	8000
MS	0.91	0.25	0.95	0.45	0.99	0.72
AT	0.93	0.95	0.91	0.66	0.90	0.95
BI	0.75	0.95	0.92	0.98	0.97	0.97
Means	0.96	0.93	0.95	0.86	0.99	0.94

Wier, Jesteadt, and Green (1977)

	Frequency (Hz)							
	200	400	600	800	1000	2000	4000	8000
Means	0.99	0.96	0.94	0.95	0.98	0.92	0.95	0.81

Harris (1952)

	Frequency (Hz)					
	125	250	500	1000	2000	4000
CK	0.80	0.94	0.91	0.91	0.95	0.13
JD	0.97	0.96	0.79	0.93	0.82	0.60
SE	0.96	0.91	0.93	0.96	0.98	0.92
Means	0.94	0.99	0.90	0.98	0.97	0.98

linear in d'/Hz (Rabinowitz, 1970; Jesteadt and Bilger, 1974; Jesteadt and Sims, 1975; Turner and Nelson, 1982), 4AFC-equivalence was achieved for the Wier *et al.* DLFs by multiplying their DLFs by $1.49/0.78 = 1.91$. Although the DLFs for individual listeners were not available for analysis, the coefficients of correlation given in Table II demonstrate the excellent fit achieved at all test frequencies for their mean data. At seven test frequencies Eq. (1) accounts for more than 92% of the variance, at 8000 Hz only 81% was accounted for.

The results of the least-squares analysis of the DLF intensity functions in the Harris (1952) study are also shown in Table II. Again, the mean DLF intensity functions are well fit by Eq. (1), as evidenced by the coefficients of correlation that ranged between 0.90 and 0.99 for different frequencies. With the exception of listener CK at 4000 Hz, the coefficient of correlation in Table II indicates that Eq. (1) also did an excellent job of describing DLF intensity functions from individual listeners in the Harris study.

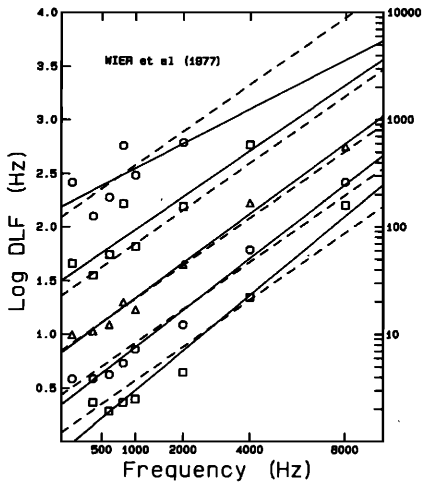

FIG. 4. Average DLFs as a function of stimulus frequency from the Wier *et al.* (1977) study, replotted here as 4AFC-equivalent DLFs (see text). Curves are arranged ordinally by sensation level (5, 10, 20, 40, and 80 dB), with the 5-dB curve at the top.

B. Stimulus frequency

Wier *et al.* (1977) have already shown that DLFs at any particular SL can be described best in the frequency domain with a square-root transformation of frequency. Figures 3–5 show DLFs plotted as a function of test frequency on their square-root scale, with SL as the parameter. For clarity, each curve is displaced from the next by a factor of two. Figure 3 contains DLFs from the present study. Figure 4 shows DLFs from Wier *et al.* (1977), and Fig. 5 shows DLFs from Harris (1952). DLFs from the latter two studies have been corrected for differences in performance level.[1] With these coordinates, the logarithm of the DLF appears as a linear function of the square root of frequency. The equation describing this function, first proposed by Wier *et al.* (1977), is given here as Eq. (2). The solid lines in Figs. 3–5 are the least-squares best-fit curves using Eq. (2). A simple visual examination of those curves indicates that the equation fits the data exceptionally well in two of the studies, ours and Harris'; it fits the data moderately well in the Wier *et al.* (1977) study.

$$\log \mathrm{DLF} = a\sqrt{F} + b. \qquad (2)$$

More objective evidence of goodness of fit for Eq. (2) is given in Table III, which shows the least-squares estimates of the parameters in Eq. (2) for each of the three studies. All the coefficients of correlation (r^2) in Table III for our data are

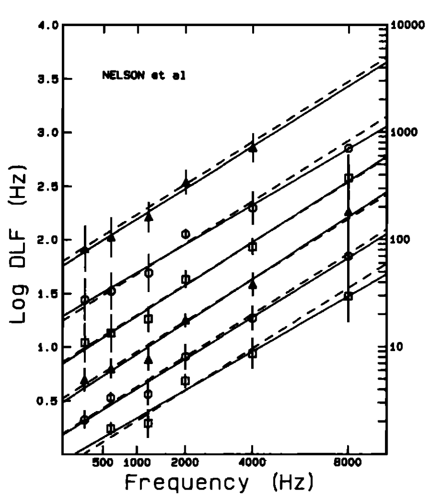

FIG. 3. Average DLFs as a function of stimulus frequency from normal-hearing listeners in the present study. The solid curves are the best-fit curves at each sensation level using Eq. (2), essentially the Wier *et al.* (1977) equation. The dashed curves are the DLFs predicted by the general equation, Eq. (4), with the constants given in Table V. The curves are arranged ordinally by sensation level (10, 20, 30, 40, 50, and 60 dB), with the curve for the lowest sensation level at the top of the graph. To avoid superposition, each curve is displaced from its neighbor by a factor of two in the DLF dimension. The actual numeric values shown on the vertical axis are applicable only for the lowest curve shown (in this case for the 60 dB SL curve). The actual DLFs for the other curves can be obtained by successive factoring by two.

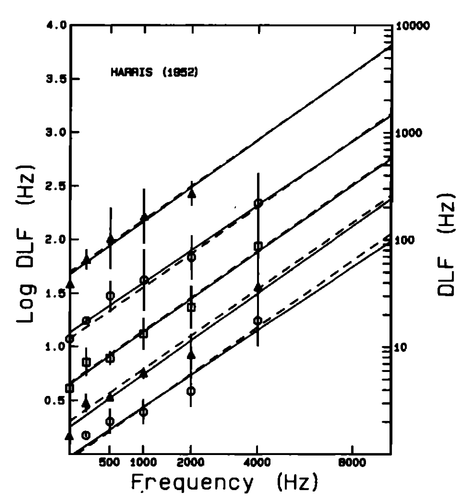

FIG. 5. Average DLFs as a function of stimulus frequency from the Harris (1952) study, replotted here as 4AFC-equivalent DLFs (see footnote 1). Curves are arranged ordinally by sensation level (5, 10, 15, 20, and 30 dB), with the 5-dB curve at the top.

TABLE III. Least-squares estimates for the parameters in Eq. (2) at each stimulus sensational level. For comparison $r^2_{\log F}$ is given for a fit using a logarithmic transformation of frequency.

SL	a	b	r^2	$r^2_{\log F}$
Nelson, Stanton, and Freyman				
10	0.023	0.27	0.998	0.933
20	0.021	0.12	0.991	0.925[a]
30	0.022	0.00	0.994	0.922[a]
40	0.022	−0.07	0.989	0.897
50	0.021	−0.06	0.995	0.928[a]
60	0.020	0.02	0.986	0.950
Total			0.993	
Wier, Jesteadt, and Green (1977)				
5	0.019	0.77	0.877	0.818
10	0.022	0.38	0.926	0.865[a]
20	0.025	−0.06	0.986	0.902[a]
40	0.026	−0.25	0.981	0.867[a]
80	0.028	−0.42	0.967	0.865
Total			0.955	
Harris (1952)				
5	0.024	0.20	0.957	0.999[a]
10	0.023	−0.03	0.984	0.943
15	0.024	−0.22	0.981	0.864[a]
20	0.024	−0.32	0.964	0.905
30	0.023	−0.28	0.950	0.879
Total			0.969	

[a] $r^2_{\log F}$ is significantly different from r^2 ($p = 0.05$).

greater than 0.98, indicating that Eq. (2) accounts for more than 98% of the variance at every SL investigated in the present study. In the other two studies, Eq. (2) also provides an excellent fit, although not quite as good as in the present study. Notice also in Table III that r^2 was generally larger when $\sqrt{F}$ rather than $\log(F)$ transformations of frequency were made. This also applies to Wier's data and to Harris' data (with the one exception in Harris' data at 5 dB SL).

C. General prediction equation

An examination of the least-squares parameters in Table III indicates that the slopes (a) in Eq. (2) are relatively constant over a wide range of SLs. This is particularly obvious for our data and for the Harris data; that trend is not so clear in the Wier *et al.* data. For the purpose of deriving a general equation to describe normal DLFs over a wide range of frequencies and intensities, it seemed reasonable to assume that slope estimates (a) in Eq. (2) are constant with SL and that the primary term in Eq. (2) that varies with SL is the intercept (b). To test those assumptions further, data from all three studies were subjected to a parallel least-squares fitting procedure (Seber, 1977).[2] For the data from each study, that procedure yielded a slope term (a') common to all SLs, intercepts (b') appropriate to each SL, and an F ratio to indicate whether or not the parallel-fit model was significantly different from an individual-fit model. The results, given in Table IV, indicate that this was not the case, i.e., a common slope (a') could reasonably be used at all SLs. The coefficients of correlation obtained with the parallel-fit model are given in column 3 of Table IV. A comparison of r^2 in Table III with R^2 (p.f.) in Table IV provides an indication of the percentage loss in variance accounted for by assuming a common slope at all SLs. Only at 5 and 80 dB SL in the Wier *et al.* data was there more than a 2% loss in the variance accounted for at any SL when the slope term was held constant.

Having demonstrated that the data could be well described by Eq. (2) with a common slope assumption, the next step was to arrive at a satisfactory description of the intercepts (b') that were derived with the parallel fit model. Those intercepts are given in Table IV, column 2, as a function of SL. Because Eq. (1) adequately described DLF changes with stimulus intensity at each test frequency, both for individual DLFs and for group mean DLFs, Eq. (1) was also regarded as an appropriate choice for describing changes in the b' intercepts that occur with changes in SL. Therefore, Eq. (1) was rewritten as Eq. (3), and least-square parameter estimates were determined for k' and m'

$$b' = k' + m'(\mathrm{SL}^{-1}). \tag{3}$$

From this point, specification of a general equation describing DLFs as a function of stimulus sensation level and

TABLE IV. Variance accounted for by the parallel-fit model and by the final prediction equation. Values of b' are the intercepts of Eq. (2) obtained with a parallel-fitting procedure that assumed a common slope a' at every sensation level. Coefficients of correlation (R^2) are given for the parallel fits (p.f.) and for the prediction-equation fits [Eq. (4)].

SL	b'	R^2(p.f)	R^2 [Eq. (4)]
Nelson, Stanton, and Freyman			
10	0.347	0.994	0.994
20	0.125	0.991	0.989
30	0.026	0.994	0.994
40	−0.029	0.988	0.988
50	−0.062	0.995	0.994
60	−0.066	0.975	0.975
Total		0.990	0.989
$a' = 0.0214$ $F = 1.40(5, 23$ d.f.); not significant ($p < 0.25$)			
Wier, Jesteadt, and Green (1977)			
5	0.570	0.815	0.804
10	0.303	0.916	0.871
20	−0.009	0.984	0.983
40	−0.156	0.973	0.971
80	−0.228	0.944	0.940
Total		0.936	0.926
$a' = 0.0238$ $F = 2.95(4, 29$ d.f.); not significant ($p < 0.025$)			
Harris (1952)			
5	0.218	0.956	0.955
10	−0.044	0.983	0.965
15	−0.209	0.981	0.981
20	−0.304	0.963	0.955
30	−0.306	0.949	0.948
Total		0.968	0.962
$a' = 0.0235$ $F = 0.01(4, 19$ d.f.); not significant ($p < 0.75$)			

TABLE V. Estimates of the parameters in the general prediction equation, Eq. (4). Total variance accounted for by Eq. (4), across all levels and frequencies, is given by Total R^2.

	k'	m'	a'	Total R^2
Nelson *et al.*	−0.15	5.056	0.0214	0.990
Wier *et al.* (1977)	−0.24	4.295	0.0238	0.926
Harris (1952)	−0.43	3.298	0.0235	0.962

test frequency is straightforward. Substitution of Eq. (3) for the b intercept in Eq. (2), and substitution of the common slope a' for the slope a in Eq. (2) yields Eq. (4), the general prediction equation. The specific values arrived at for the constants k', m', and a' are given in Table V for all three studies

$$\log \mathrm{DLF} = a'\sqrt{F} + k' + m'(\mathrm{SL}^{-1}). \qquad (4)$$

D. Accuracy of the prediction equation

The accuracy with which Eq. (4) can predict actual DLFs as a function of frequency is illustrated in Fig. 3, for the current data. The dashed curves in Fig. 3 are the DLFs predicted by Eq. (4). In this case, recall that both frequency and SL are independent variables, and a', k', and m' are fixed constants. From a subjective viewpoint, these curves provide a very good description of the actual data, all points lying on or near to the lines of best fit.

The more objective assessment of predictive accuracy, given by the coefficients of correlation in Table IV, confirms these observations. The coefficients, R^2 [Eq. (4)], indicate that between 97% and 99% of the variance was still accounted for at each SL by the general prediction equation. At 60 dB SL a somewhat poorer fit resulted ($R^2 = 0.975$), but one still acceptable by most standards. The total variance accounted for by the prediction equation, across all levels and frequencies, was 98.9%. It appears that Eq. (4), the general prediction equation, provides an accurate description of frequency-difference thresholds over a wide range of stimulus levels (10–60 dB SL) and stimulus frequencies (300–8000 Hz).

Figure 4 illustrates, with the dashed curves, similar comparisons for the Wier *et al.* (1977) data. Their data were not as well fit by Eq. (4) as were the present data. Table IV shows that 80% to 98% of the variance was accounted for by Eq. (4) at different SLs. The total variance accounted for was 93%. Considering the amount of variability in the data, we consider Eq. (4) to provide an adequate approximation of these frequency discimination data.

Figure 5 shows the least-square fits of data obtained by Harris (1952). Table IV shows that 95% to 98% of the variance was accounted for by Eq. (4) at different SLs. The total variance accounted for over all SLs and frequencies was 96%.

The major differences between these three studies occurred in the behavior of the a slopes of Eq. (2) as a function of SL. If the analyses of the present data and the Harris data were not available, one would be tempted to propose a more complex equation than Eq. (4), one that would account for the changes in a slopes of Eq. (2) with SL that are apparent in the Wier *et al.* data. However, given evidence of relatively constant a slopes in two out of three studies, it appears appropriate to make the assumption of a constant slope at all SLs, at least for the purposes of attempting a general equation.

III. CONCLUSIONS

The general prediction equation, Eq. (4), accounts for 99% of the total variance in the present data, 96% in the Harris (1952) data, and 93% in the Wier *et al.* (1977) data. We conclude that the general prediction equation provides an approximation of DLFs over a wide range of SLs and test frequencies that is essentially as good as the original equation, Eq. (2), suggested by Wier *et al.*

Although the physiological relevance of either the $\sqrt{F}$ transformation of stimulus frequency or the SL^{-1} transformation of stimulus level has not yet been addressed, both transformations yield linear dimensions that facilitate a description of frequency discrimination from normal-hearing listeners over a wide range of stimulus frequencies and stimulus levels. That description might prove to be useful for defining frequency-discrimination deficits in hearing-impaired listeners as ratios between the obtained DLF from an impaired ear and the predicted DLF from normal ears. However, any general use of Eq. (4) for specifying "norms" for frequency discrimination must include some definition of dispersion in a group of listeners who are acceptably defined as being "normal." That is not the purpose of this paper. Estimates of dispersion must await data collection from a larger group of listeners.

Finally, it should be noted that the prediction equation does not include any terms for describing changes in frequency discrimination that occur with stimulus duration. Although some previous evidence suggests that DLFs might change with the square root of duration (Liang and Chistovich, 1961), preliminary evidence in our laboratory suggests that the duration effect is dependent upon SL. We believe further investigation of this dependent relationship is warranted before attempting to include duration effects in a general prediction equation.

ACKNOWLEDGMENTS

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[1] As with the Wier *et al.* (1977) data, the Harris (1952) data were also converted to 4 AFC equivalent DLFs by multiplying each DLF by 1.91. Harris used a variant of the constant-stimulus two-interval forced-choice technique (AX), so the validity of that conversion is open to question. In addition, Harris employed 1.4-sec test tones, while the Wier *et al.* study employed 500-msec tone bursts and the present study employed 300-msec tone bursts. This difference in test-tone duration may explain why Harris obtained more acute DLFs than either of the other two studies. We have made no attempts here to account for signal duration effects, except to note that Harris used longer signals and obtained lower DLFs, which is in the proper direction (Doughty and Garner, 1948; Liang and Chistovich, 1961; Moore, 1973; Ronken, 1971). More data on duration effects and their dependencies on signal level are required before including them in the general prediction equation.

[2]The object of the parallel fitting procedure was to obtain a linear regression equation to fit log DLF versus square-root frequency using one common slope term for all SLs. Seber's (1977) procedure also tests whether the parallel-fit model, with a common slope, is significantly different from a model that allows the slope to vary at each SL. In the parallel-fit model, the slope is obtained by minimizing the sum of the squared residuals,

$$\mathrm{RSS} = \Sigma_k \Sigma_i (Y_{ki} - b_k - aX_{ki})^2,$$

where k represents the different SLs and i represents the different test frequencies. Minimization is accomplished by setting partial derivatives equal to zero and solving for a and b. The common slope becomes:

$$a' = \frac{\Sigma_k \Sigma_i (Y_{ki} - \overline{Y}_{k.})(X_{ki} - \overline{X}_{k.})}{\Sigma_k \Sigma_i (X_{ki} - \overline{X}_{k.})^2}.$$

When the common slope is used, the intercept for each SL then becomes:

$$b'_k = \overline{Y}_{k.} - a'\overline{X}_{k.}.$$

The values of b'_k and a' are substituted into the initial RSS equation to obtain the sum of squared residuals for the parallel-fit model (RSSpf). RSSpf is then compared to the sum of squared residuals that is obtained when each data set is fit separately (RSS). The resulting F statistic is given by:

$$F = \frac{(\mathrm{RSSpf} - \mathrm{RSS})/(K-1)}{\mathrm{RSS}/(N-2K)},$$

where K is the number of SLs and N is the total number of x,y pairs across all SLs. The F ratio has $K-1$ and $N-2K$ degrees of freedom.

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